

Quiz 9

September 30, 2016

Show all work and circle your final answer.

1. Compute $f''(e)$ if $f(x) = \ln(\ln x)$. Note: $\frac{d}{dx} (\ln x) = \frac{1}{x}$.

$$f'(x) = \frac{1}{\ln x} \cdot \frac{d}{dx} (\ln x) = \frac{1}{\ln x} \cdot \frac{1}{x} = (x \ln x)^{-1} \quad \text{by chain rule}$$

$$\begin{aligned} f''(x) &= -1 (x \ln x)^{-2} \frac{d}{dx} (x \ln x) \\ &= -1 (x \ln x)^{-2} \left(\ln x + x \cdot \frac{1}{x} \right) \\ &= \frac{-(\ln x + 1)}{(x \ln x)^2} \end{aligned}$$

$$f''(e) = \frac{-(\ln e + 1)}{(e \ln e)^2} = \boxed{\frac{-2}{e^2}}$$

2. Let $f(x)$ and $g(x)$ be differentiable functions. Let $h(x) = (f[g(x)])^3$. Compute $h'(1)$ using the following table:

	1	2	3	4
$f(x)$	1	2	5	1
$f'(x)$	4	3	2	1
$g(x)$	3	-2	4	-5
$g'(x)$	-1	3	-2	3

$$\begin{aligned} h'(x) &= 3(f[g(x)])^2 \frac{d}{dx} (f[g(x)]) \\ &= 3(f[g(x)])^2 \cdot f'(g(x)) \cdot g'(x) \end{aligned}$$

$$\begin{aligned} h'(1) &= 3(f[g(1)])^2 \cdot f'(g(1)) \cdot g'(1) \\ &= 3(f(3))^2 \cdot f'(3) \cdot (-1) \end{aligned}$$

$$= 3(5)^2 \cdot 2 \cdot (-1) = \boxed{-150}$$

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